arXiv: 2207.07873 "Mimetic Curvaton"

Curvaton Mechanism.

Convert entropy perturbation into curvature perturbation in radiation/matter-dominated universe.

$$\zeta = -H\frac{\delta\rho}{\dot{\rho}} \quad , \quad \dot{\rho} + 3H(\rho + \cancel{P}\, w\rho) = 0$$

$$-3H(1+w_{total})\rho_{total} = \dot{\rho}_{total} = \sum_i \dot{\rho}_i$$

$$= \sum_i -3H(1+w_i)\rho_i$$

$$\zeta = \frac{\sum_i (1+w_i)\rho_i\zeta_i}{\sum_i (1+w_i)\rho_i} = \sum_i r_i \delta_i$$

$$r_i = \frac{\rho_i}{(1+w_{total})\rho_{total}} \quad , \quad \delta_i = \frac{\delta\rho_i}{\rho_i}$$

If there is no transformation from one component of energy into another, the curvature perturbation depends on δ_i s.

Assume perturbation of inflation vanished

Curvaton field σ

$$\mathcal{L}_\sigma = \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}(\nabla\sigma)^2 - V(\sigma)$$

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$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0$$

$\delta\sigma$ is decoupled from other perturbation

$$\ddot{\delta\sigma}_k + 3H\dot{\delta\sigma}_k + (k^2/a^2 + V_{\sigma\sigma})\delta\sigma_k = 0$$

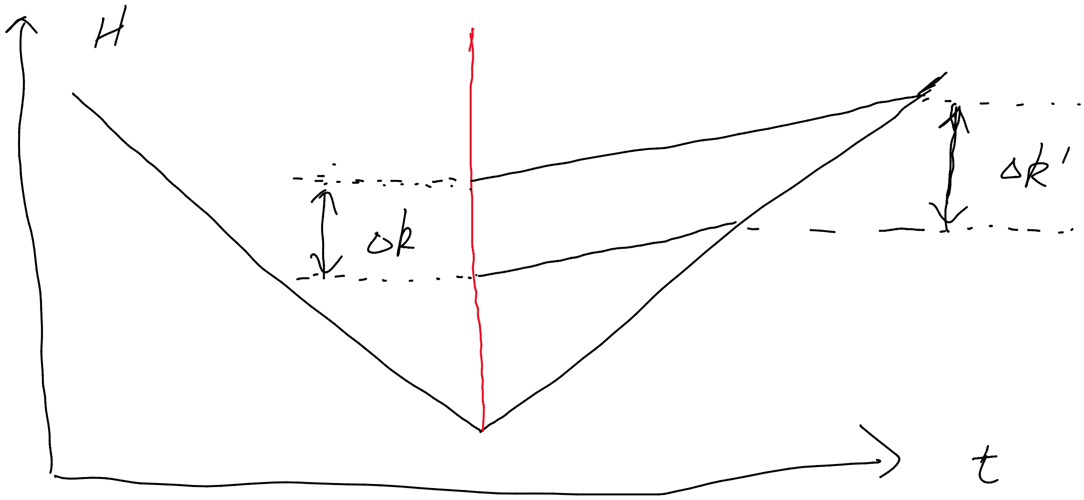
assume $|V_{\sigma\sigma}| \ll H^2$

super-horizon $\quad \ddot{\delta\sigma}_k + 3H\dot{\delta\sigma}_k + V_{\sigma\sigma}\delta\sigma_k = 0$

$V = \frac{1}{2}m_\sigma^2\sigma^2$, σ & $\delta\sigma$ are both oscillating

$$\delta_\sigma = \frac{\delta\rho_\sigma}{\rho_\sigma} \propto \frac{2\langle\sigma\delta\sigma\rangle + \langle(\delta\sigma)^2\rangle}{\langle\sigma^2\rangle + 2\langle\sigma\delta\sigma\rangle + \langle(\delta\sigma)^2\rangle}$$

δ_σ is independent on time on super-horizon.



$\Delta k \neq \Delta k'$, not scale-invariance any more

$$P_k \propto \frac{1}{k^{3+a}}$$

Mimetic gravity : $g_{\mu\nu} = (\tilde{g}^{\alpha\beta}\partial_\alpha\Phi\,\partial_\beta\Phi)\tilde{g}_{\mu\nu}$

$\tilde{g} \to \Omega^2\tilde{g}$

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$$\tilde{g}_{\mu\nu} \to \Omega^2 \tilde{g}_{\mu\nu} \quad , \quad g_{\mu\nu} \to g_{\mu\nu}$$

Choose Ψ to express metric.

EoM of $\tilde{g}_{\mu\nu}$ and Ψ contribute a new term on $T_{\mu\nu}$

$$G_{\mu\nu} - T_{\mu\nu} - (G-T)\partial_\mu\Psi\partial_\nu\Psi = 0, \quad G = G^\mu{}_\mu$$

On this aspect, the equivalent action is

$$S = \int \sqrt{-g}\, d^4x \left[\lambda\left(\partial^\mu\Psi\partial_\mu\Psi + \Omega\right) + \frac{M_p^2}{2}R - V(\Psi)\right]$$

λ is a Lagrang multipler.

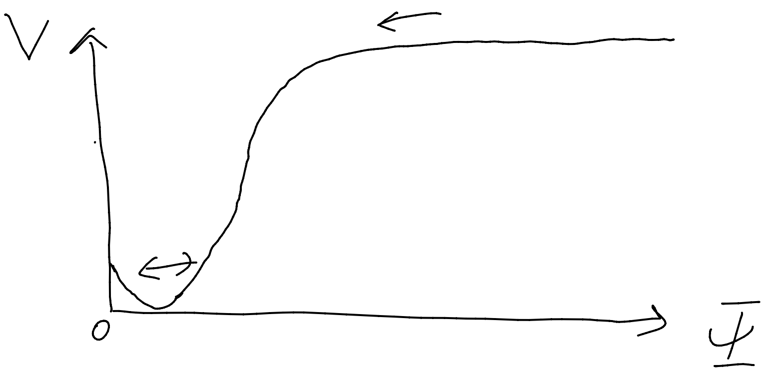
For isotropic and homogeneous field.

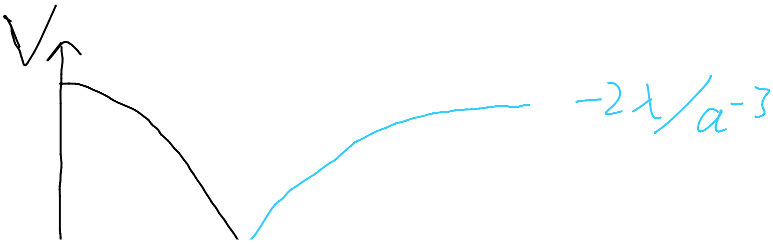
$$3M_p^2H^2 = V - 2\lambda\Omega \quad , \quad 2M_p^2\dot{H} = 2\lambda\Omega$$

$$\Rightarrow \dot{\lambda} + 3H\lambda - \frac{\dot{V}}{2\Omega} = 0 \Rightarrow \lambda = \frac{1}{2\Omega}a^{-3}\int_{t_i}^{t} dt\, a^3\dot{V}$$

V control the behaviours of λ

To inflation (large field inflation as example)





$$\frac{1}{2}\dot{\Psi}^2 + V(\Psi) \Longrightarrow -2\lambda + \Omega_r + \text{entropy field}$$

Single field: $\delta\ddot{\Psi} + H\delta\dot{\Psi} + \cdots = 0$ since $\delta\dot{\Psi} = \Phi$

irrelevant of wavelength

Multi-fields: $g_{\mu\nu} = (G_{ab}\tilde{g}^{\alpha\beta}\partial_\alpha\phi^a\partial_\beta\phi^b)\tilde{g}_{\mu\nu}$

$$\dot{u}_T = \varepsilon H u_T + \dot{\theta} u_N$$

u_T is adiabatic perturbation, u_N is entropy ...

kinetic term of u_T gets lost.

Mimetic Curvaton

Inflaton-like field φ, curvaton-like field θ.

$$S = \int \sqrt{-g}\, d^4x \left\{ \frac{M_p^2}{2} R + \lambda\left[-\frac{1}{2}\dot{\varphi}^2 - 3\sinh\left(\frac{\varphi}{\sqrt{6}M}\right)\dot{\theta}^2 + M^4\right] - V(\varphi,\theta) \right\}$$

$\varphi = \varphi_0 + \delta\varphi$, $\theta = \theta_0 + \delta\theta$, $V(\varphi,\theta) = V(\varphi)$

$$\begin{cases} \ddot{\varphi} + \left(3H + \frac{\dot{\lambda}}{\lambda}\right)\dot{\varphi} - \frac{\sqrt{6}}{M}\sinh\left(\frac{\varphi}{\sqrt{6}M}\right)\cosh\left(\frac{\varphi}{\sqrt{6}M}\right)\dot{\theta}^2 + \frac{1}{\lambda}\frac{\partial V}{\partial\varphi} \simeq 0 \\ \ddot{\theta} + \left(3H + \frac{\dot{\lambda}}{\lambda}\right)\dot{\theta} + \sqrt{\frac{2}{3M^2}}\coth\left(\frac{\varphi}{\sqrt{6}M}\right)\dot{\varphi}\dot{\theta} \approx 0 \\ \dot{\varphi}^2 + 6\sinh^2\left(\frac{\varphi}{\ }\right)\dot{\theta}^2 = 2M^4 \end{cases}$$

$$\dot{\varphi}^2 + 6\sinh^2\left(\frac{\varphi}{\sqrt{6}M}\right)\dot{\theta}^2 = 2M^4$$

After reheating, $\frac{1}{2}\dot{\varphi}^2 + V(\varphi) \to -2\lambda + \Omega_\gamma$ + entropy field

$$+\frac{1}{2}\dot{\theta}^2 + \frac{1}{2}\dot{\psi}^2$$

thus, $\dot{\psi} = \pm\sqrt{2}M^2$ satisfying the constraint.

$$\begin{cases} \ddot{\varphi}_0 - \frac{1}{\varphi_0}\left(2M^4 - \dot{\psi}^2 - \dot{\varphi}_0^2\right) + \frac{1}{\lambda}V' = 0 \\ \ddot{\theta}_0 + 2\frac{\dot{\varphi}_0}{\varphi_0}\dot{\theta}_0 = 0 \\ \dot{\varphi}_0^2 + \dot{\psi}^2 + 6\sinh^2\left(\frac{\varphi_0}{\sqrt{6}M}\right)\dot{\theta}_0^2 = 2M^4 \end{cases}$$

$$\begin{cases} \delta\ddot{\varphi} + \frac{k^2}{a^2}\delta\varphi + \frac{2M^4 - \dot{\psi}^2}{\varphi_0^2}\delta\varphi + 2\frac{\dot{\varphi}_0}{\varphi_0}\delta\dot{\varphi} + \frac{1}{2}V''\delta\varphi = 0 \\ \delta\ddot{\theta} + \frac{k^2}{a^2}\delta\theta + 2\frac{\dot{\varphi}_0\dot{\theta}_0}{\varphi_0^2}\delta\varphi + \frac{2}{\varphi_0}\left(\delta\dot{\varphi}\,\dot{\theta}_0 + \dot{\varphi}_0\,\delta\dot{\theta}\right) = 0 \end{cases}$$

At small φ_0, choose $\frac{V}{\lambda} = \frac{1}{4}m_{eff}^2\varphi_0^2 + (2M^4 - \dot{\psi}^2)\ln\left(\frac{\varphi_0}{\varphi_1}\right)$

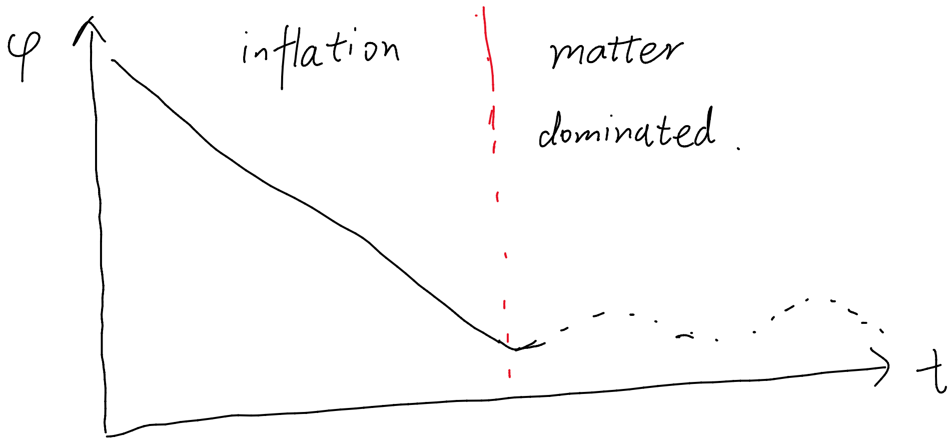
Since φ_0 doesn't vanish, V won't be divergence.

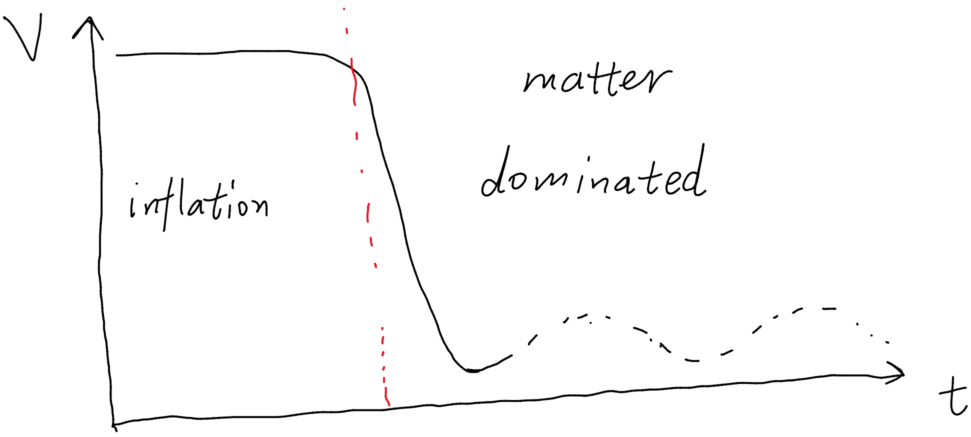
$$\frac{d^2}{dt^2}(\varphi_0^2) + m_{eff}^2\varphi_0^2 = 0$$

$$A m_{eff}^2 \ll H^2 \Rightarrow A m_{eff}^2 t^2 \ll 1$$

$$\begin{cases} \varphi_0^2 = A\sin(m_{eff}t) \simeq A m_{eff} t, \quad \dot{\theta}_0 = \frac{A m_{eff}}{2\sqrt{3}\varphi_0^2} \simeq \frac{1}{2\sqrt{3}t} \\ \delta\varphi = \frac{1}{\varphi_0}\left(D_+ e^{ik\tau} + D_- e^{ik\tau}\right) \end{cases}$$

$$\begin{cases} \varphi_0^2 = A\sinh(m_{eff}t) \simeq A m_{eff} t, \quad \dot{\theta}_0 = \dfrac{A m_{eff}}{2\sqrt{3}\,\varphi_0^2} \simeq \dfrac{1}{2\sqrt{3}\,t} \\ \delta\varphi = \dfrac{1}{\varphi_0}\left(D_+ e^{ik\tau} + D_- e^{ik\tau}\right) \\ \delta\theta = \dfrac{\sqrt{3}}{3A m_{eff} t}\left(D_+ e^{ik\tau} + D_- e^{ik\tau}\right) \end{cases}$$





$$\delta_\varphi = \frac{\delta\rho_\varphi}{\rho_\varphi}, \quad \rho_\varphi \sim m_{eff}^2 \varphi_0^2, \quad \delta\rho_\varphi \sim m_{eff}^2 \langle \varphi_0 \delta\varphi_0 \rangle$$

$\delta_\varphi \propto \dfrac{1}{t^2}$ is negligible

$$\delta_\theta = \frac{\delta\rho_\theta}{\rho_\theta}, \quad \rho_\theta \sim \varphi_0^2 \dot{\theta}_0^2, \quad \delta\rho_\theta \sim \varphi_0^2 \langle \dot{\theta}_0 \delta\dot{\theta}_0 \rangle = \varphi_0^2 \sqrt{\dot{\theta}_0^2 \langle \delta\dot{\theta}^2 \rangle}$$

$$\delta\theta = \frac{1}{t}F(t), \quad \delta\dot{\theta} = -\frac{1}{t^2}F(t) + \frac{1}{t}F'(t) \approx \frac{1}{t}F'(t)$$

$\langle \mathcal{F}'(t)^2 \rangle = \langle \mathcal{F}(t)^2 \rangle$ for oscillators.

$\langle \delta\dot{\theta}^2 \rangle \simeq \langle \delta\theta^2 \rangle \Rightarrow \delta_\theta$ is independent on time.

$$\zeta = \sum_i r_i \delta_i$$

$\Rightarrow$ We convert the entropy perturbation into curvature perturbation.

MiMetic gravity can describe inflation with curvaton mechanism.

Outlook:

① More general field space metric

② Bounce cosmology

③ Maybe a new PBH generating process about $\delta\lambda$.